

Editorial Manager(tm) for Journal of Bridge Engineering  
Manuscript Draft

Manuscript Number: BEENG-573R1

Title: Moment and Shear Load Distribution Factors for Multi-girder Bridges Subjected to Overloads

Article Type: Technical Paper

Corresponding Author: HAN UG BAE, Ph.D.

Corresponding Author's Institution: University of Wisconsin-Madison

First Author: HAN UG BAE, Ph.D.

Order of Authors: HAN UG BAE, Ph.D.; Michael G. Oliva, Ph.D.

**Abstract:** Movement of industrial freight infrequently requires special overload vehicles weighing 5 to 6 times the normal legal truck weights. The gross vehicle weight of the vehicles frequently exceeds 1800 kN while the normal interstate legal limit in the U.S. is 356 kN. Because of the unusual configuration of the vehicles it is difficult to analyze the effect of these loads on highway bridges using current simplified analysis methods. This report aims to provide modified moment and shear load distribution factor equations for the vehicles to quickly determine their effects on multi girder bridges. Finite element analyses of 118 multi-girder bridges and 16 load cases of overload vehicles for each multi-girder bridge were performed and the load distribution factor equations for the multi-girder bridges were proposed based on the analysis results. Various configurations of the vehicles, number of bridge spans, skew angles of the bridge and diaphragms were considered in developing the equations. The developed equations were found to be capable of replacing a time consuming 3D finite element analysis rationally and conservatively.

Suggested Reviewers:

Opposed Reviewers:

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

# Moment and Shear Load Distribution Factors for Multi-girder Bridges Subjected to Overloads

Han Ug Bae<sup>1</sup>; and Michael G. Oliva<sup>2</sup>

**Abstract:** Movement of industrial freight infrequently requires special overload vehicles weighing 5 to 6 times the normal legal truck weights. The gross vehicle weight of the vehicles frequently exceeds 1800 kN while the normal interstate legal limit in the U.S. is 356 kN. Because of the unusual configuration of the vehicles it is difficult to analyze the effect of these loads on highway bridges using current simplified analysis methods. This report aims to provide modified moment and shear load distribution factor equations for the vehicles to quickly determine their effects on multi girder bridges. Finite element analyses of 118 multi-girder bridges and 16 load cases of overload vehicles for each multi-girder bridge were performed and the load distribution factor equations for the multi-girder bridges were proposed based on the analysis results. Various configurations of the vehicles, number of bridge spans, skew angles of the bridge and diaphragms were considered in developing the equations. The developed equations were found to be capable of replacing a time consuming 3D finite element analysis rationally and conservatively.

---

<sup>1</sup>Postdoctoral Fellow, Dept. of Civil and Environmental Engineering, Univ. of Wisconsin-Madison, 1415 Engineering Dr., Madison, WI 53706.

<sup>2</sup>Professor, Dept. of Civil and Environmental Engineering, Univ. of Wisconsin-Madison, 1415 Engineering Dr., Madison, WI 53706.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

**Keywords:** Overweight trucks; Oversize trucks; Load distribution factors; Multi-girder bridges; Bridge analysis; Finite element analysis; Bridge loads; Skewed bridges.

## Introduction

Movement of industrial freight infrequently requires special overload vehicles weighing 5 to 6 times the normal legal truck weight to move across highway systems. Fig. 1 shows one example of a special overload vehicle. The gross vehicle weight of these superload vehicles frequently exceeds 1800 kN while the normal interstate legal limit for gross vehicle weight in the United States is 356 kN (Tabsh and Tabatabai 2001). Examples of the loads carried by the vehicles are pressure vessels and transformers used in power plants, wind turbine components, boilers, military hardware, beams and barges.

It is necessary for transportation agencies to analyze bridges subjected to these types of vehicles to provide special permits along a specified pathway. Because of the unusual configuration of the vehicles, time consuming 3 dimensional finite element analysis may be required to evaluate the effect of the vehicles on highway bridges since simple analysis methods are not well established and the possibility of errors in estimating the impact of the loads on these structures could affect safety. The Wisconsin Department of Transportation was particularly interested in using a simple but practical method to estimate girder forces under these loads for the permitting process. The work described here provides simplified empirical based analysis method to evaluate forces in common highway bridges subjected to these vehicles.

Current AASHTO LRFD Bridge Design Specifications (2009) that are used for design and rating of bridges are based on a design truck that is intended to represent the effects of a range of normal trucks. Engineers are hesitant to use prescriptions for standard load analysis methods

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 from those design specifications for super overloads because they may not be applicable with the  
2 specially configured overload vehicles because of different axle and wheel configurations.

3 Previous research studies have focused on the analysis and assessment of bridges subjected to  
4 overload vehicles. Hays (1984) developed a procedure to find moment load distribution factors  
5 for prestressed simple span bridges subjected to overloads using linear finite element analysis. In  
6 his research, influence surfaces of the girder centerline moments were obtained for a specific  
7 bridge and then a load distribution factor for the bridge was calculated based on the influence  
8 surfaces. Chou (1996) studied the effect of overloads on bridge design and found that the axle  
9 load ratios for various types of overloads were different from the ratios for the design truck.  
10 Tabsh and Tabatabai (2001) developed modification factors for the AASHTO flexural and shear  
11 load distribution factors applicable to simple span bridges without skew to account for oversized  
12 trucks. The variables affecting the factors were girder spacing and lateral spacing of wheels. Fu  
13 and Hag-Elsafi (1996a) presented a live load model including overloads for assessing highway  
14 bridge's structural reliability. Fu and Hag-Elsafi (1996b) developed an overload permit checking  
15 procedure for bridge evaluation, in the format of load and resistance factors. Chou et al. (1999)  
16 presented a technique utilizing the combined effect of truck gross weight, axle loads and  
17 spacings to assess overload's effects on highway bridges. The scope of previous research,  
18 however, was often limited to vehicles weighing less than 1800 kN and the focus was also  
19 usually limited to the single lane trailer or the simply supported bridges without skew.

20 In this research, new moment and shear load distribution factor equations for various types of  
21 oversize overweight vehicles, including single lane trailers and dual lane trailers (Fig. 2), were  
22 developed. The load distribution factor is a key parameter in quickly analyzing the effects of the  
23 vehicles on multi-girder bridges without performing time consuming 3-dimensional finite

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 element analyses. Finite element analyses of 118 multi-girder bridges with 16 load cases of  
2 oversize overweight vehicles for each multi-girder bridge were performed and simple load  
3 distribution factor equations for the multi-girder bridges are proposed based on the analysis  
4 results. The various configurations of vehicles, number of bridge spans, skew angles of the  
5 bridge and existence of diaphragms between girders were all considered in developing the  
6 equations (Fig. 2 and Table 1). Representative analysis results are presented in this paper and  
7 complete analysis results are presented in a separate report (Bae and Oliva 2010).

8

### 9 **Prototype Bridges**

10 118 prototype bridges with different configurations were selected and analyzed under oversize  
11 overweight vehicle loads. The variables for the configurations of the bridges were length of span,  
12 spacing of girders, depth of the deck, type of girder, skew angle, number of spans and end  
13 diaphragms. The selected sets of prototype bridge parameters are summarized in Table 1. The  
14 prototype bridges had no intermediate diaphragms within the span since analyses of bridges  
15 without intermediate diaphragms within the span are conservative. (AASHTO 2009-C4.6.2.2.2b)  
16 The four types of girders, i.e. steel girder type 1, steel girder type 2, concrete I girder and wide  
17 flange concrete girder, were selected to consider different ranges of stiffness and geometry. The  
18 dimensions of the selected girders are shown in Fig. 3.

19 Each bridge was assumed to have five girders. Four girder bridges were not examined because  
20 they make up only 2.9% of the national bridge inventory and their inclusion would provide more  
21 conservative results. Identical girder spacings were used with the different girders in the analyses.  
22 The development of the load distribution factor equations for oversize overweight vehicles based  
23 on the analyses of five girder bridges is conservative for bridges with five or more girders since

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 the load distribution factors for bridges with five girders is higher than for bridges with six or  
2 more girders. The analyses were focused on finding the load distribution factors of the first  
3 interior girder adjacent to the exterior girder where the load distribution factors are generally the  
4 largest (Tabsh and Tabatabai 2001). Adding more interior girders would have little effect  
5 (Bishara et al. 1993). Load distribution factors for exterior girders were excluded in the  
6 development of the load distribution factor equations since they can be calculated using a simple  
7 lever rule and they are highly dependent on the length of the roadway overhang. The position of  
8 the overload vehicle can also be controlled during crossing to minimize the impact on exterior girders.  
9 Detailed configurations of the selected bridges are described in Table 2.

**11 Oversize Overweight Vehicles**

12 A representative set of overload vehicles in appropriate configurations was needed to conduct the  
13 analyses. Initial information on the configuration of overload vehicles was collected from major  
14 carriers in the United States and the Wisconsin Department of Transportation. There were two  
15 major types of oversize overload vehicles described, i.e. single lane trailers and dual lane trailers.  
16 Transverse wheel spacings of the truck trailers selected for the analyses are shown in Fig. 2. The  
17 spacings shown in the figure may vary by a couple of inches depending on the trailer type. This  
18 variation would not significantly affect the analysis results. The transverse spacing between the  
19 centers of the middle dual wheels for a dual lane trailer were selected as 610 mm, 1829 mm and  
20 3048 mm for the analysis.

21 The representative configuration and longitudinal axle spacing of the vehicles were selected  
22 based on collected overload vehicle measurement data from major transporters in the U.S. The  
23 most and the least intensive loadings in the longitudinal direction were selected for the single

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 lane trailer loading and the dual lane trailer loading cases. The longitudinal axle configurations  
2 of the selected vehicles are shown in Fig. 4.

3 The load configurations resulted in 16 load cases [2 maximum load cases (moment and shear) x  
4 2 cases of different axle spacing (as shown in Fig. 4) x 4 cases of transverse wheel spacing (1  
5 case for single lane trailer and 3 cases for dual lane trailer)].

### 7 **3D Finite Element Analysis**

#### 8 ***Verification of Analysis Technique***

9 Finite element schemes for analyzing the selected prototype bridge configurations were selected  
10 and needed to be verified. The previous load testing of a Wisconsin bridge, performed by the  
11 University of Missouri – Rolla (Conachen 2005), was used for the verification. The  
12 superstructure of the bridge consists of two continuous spans having a length of 32.81 m. The  
13 cross section consists of five prestressed concrete girders equally spaced, supporting a reinforced  
14 concrete deck. The bridge was subjected to 2 lane truck loading at the mid-span of the first span  
15 and deflections of the five girders were measured at the mid-span of the first span.

16 Two modeling schemes using SAP2000 were used to simulate the bridge tests. The modeling  
17 schemes were 1) shell (concrete deck) + shell (steel or concrete girder) + Rigid link (to connect  
18 deck and girders) and 2) shell (concrete deck) + frame (steel or concrete girder) + Rigid link (to  
19 connect deck and girders). The results of the analyses with comparison to experimental results  
20 from the Wisconsin bridge are shown in Fig. 5.

21 The results shown in Fig. 5 indicate that finite element analyses can predict the behavior of the  
22 bridge with relatively high accuracy. The Shell (concrete deck) + Frame (steel or concrete girder)  
23 + Rigid link (to connect deck and girders) model was selected for the study since the model

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 showed an accurate result while being relatively simple. The finite element analysis technique  
2 accurately simulated the test results within 8% of the measured deflection.

3  
4 **Description of Modeling Technique**

5 All of the prototype bridges subjected to the selected vehicles remained in the linearly elastic  
6 range and nonlinearity of the materials is not considered in this study. The compressive  
7 strengths of the concrete deck and concrete girder were selected as 27.6 MPa and 55.1 MPa,  
8 respectively, commonly used design strengths. The elastic modulus of the concrete deck and  
9 concrete girder were selected as 24.8 GPa and 35.1 GPa, respectively. The elastic modulus of  
10 the steel girder was selected as 200 GPa.

11 The longitudinal and transverse locations of the vehicles on the bridge were identified to  
12 maximize moment or shear in the girder. The load distribution factor for each load case was  
13 found by dividing the moment or shear in the interior girder, adjacent to the exterior girder, by  
14 the sum of the moment or shear in all of the girders. The moments were calculated including the  
15 moments in the girders and the axial forces in the deck and girders to consider composite  
16 behavior of the deck and girders. The multiple presence factor in the AASHTO LRFD bridge  
17 design manual (2009) was assumed to be 1.00 to calculate the load distribution factor in this  
18 study assuming that the weights of the superload vehicles are well measured and controlled. The  
19 dynamic allowance in the AASHTO LRFD bridge design manual (2009) was not used to  
20 calculate the load distribution factor in this study since the velocity of the oversize overweight  
21 vehicles is expected to be less than 8 km/h.

22  
23 **Critical Parameters**



1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1     **Skew Angle**

2     Analyses of 10 single span concrete I girder bridges with skew were performed to investigate  
3     and determine the load distribution factor for bridges with skew. The analysis results for the  
4     case-8 bridge configurations in Table 2 with different skews and with and without end  
5     diaphragms are shown in the plots of Fig. 6 (moment load distribution factors) and Fig. 7 (shear  
6     load distribution factors). For all the scenarios the load distribution factors decrease as the skew  
7     angle increases which corroborates the same conclusions by Bishara et al. (1993). This was more  
8     evident for the shear load distribution factor and the dual lane trailer loading case. The results  
9     without end diaphragms showed higher load distribution factors compared to those with end  
10    diaphragms, which indicates that the result without end diaphragms will provide conservative  
11    load predictions in girders.

12  
13    **Number of Spans**

14    Analysis of 8 continuous span concrete I girder bridges without skew was performed to expand  
15    the applicability of the load distribution factor equations to general continuous span bridges. The  
16    analysis focused on the positive moment load distribution factor near the center of span and the  
17    negative moment load distribution factor near the location of the piers in continuous span bridges.  
18    The analysis results without end diaphragms are shown in the plots of Fig. 8.  
19    The positive moment load distribution factors for 2 span bridges in Fig. 8 showed less than 6.8%  
20    difference compared with those of single span bridges, while the negative moment load  
21    distribution factors of 2 span bridges were 23% lower to 52 % higher than the positive moment  
22    load distribution factors of the single span bridges.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1     **End Diaphragm**

2     The analysis results shown in Figs. 6 and 7 (bridges with skew) indicated that the analysis  
3     without an end diaphragm generally predicts higher load distribution factors than when  
4     diaphragms are used. The end diaphragms were assumed to be of solid cast-in-place concrete  
5     that are typical in Wisconsin beam bridges. Eight additional analyses of single span concrete I  
6     girder bridges without skew were performed for cases 19, 2, 8, 20, 21, 6, 22 and 23 in Table 2,  
7     with end diaphragms, for further investigation of the effects of the end diaphragms. The analysis  
8     results are shown in Figs. 9 and 10 with comparisons to the same cases without end diaphragms.  
9     The results show that the moment load distribution factors are larger without end diaphragms  
10    while the shear load distribution factors are dependent on the span length.

11    The Moment load distribution factors found from analysis of concrete girder bridges without end  
12    diaphragms were 2.9 ~ 9.6 % higher than those with end diaphragms. The shear force load  
13    distribution factors found from analysis without end diaphragms were between 7.0 % lower to  
14    7.7 % higher than those with end diaphragms. The effects of the end diaphragms, typical steel  
15    cross frames, on the load distribution factors found for steel girder bridges are smaller since the  
16    stiffness of steel end diaphragms in steel girder bridges is lower than the typical concrete  
17    diaphragms in concrete girder bridges.

18  
19     **Development of Load Distribution Factor Equations**

20    The load distribution factor equations for multi-girder bridges subjected to overload vehicles  
21    were assembled based on the results from the 118 multi-girder bridge analyses with varied bridge  
22    and loading parameters. The equations were developed under the assumption that the dynamic

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 load allowance for the overload vehicles is 0% with slow crossing speeds. It is assumed that only  
2 one overload vehicle will be on a bridge at a time.

3 The new simplified equations for load distribution factors in bridges with oversize overload  
4 vehicles were developed by curve fitting with the analysis data. These simplified methods for  
5 calculating load distribution factor are shown in Equations (1) and (2) with information from  
6 Tables 3 and 4.

$$\text{Single lane trailer: } CR S^a L^b t^c K_g^d \tag{1}$$

$$\text{Dual lane trailer: } CR S^a L^b t^c K_g^d S_w^e \tag{2}$$

9 where  $C$  = constant;  $R$  = correction factor;  $S$  = girder spacing (mm);  $L$  = span (m);  $t$  = deck depth  
10 (mm);  $K_g = n(I + Ae_g^2)$  = longitudinal stiffness parameter (mm<sup>4</sup>);  $n = E_B / E_D$ ;  $I$  = moment of  
11 inertia of girder (mm<sup>4</sup>);  $A$  = cross-sectional area of girder (mm<sup>2</sup>);  $e_g$  = distance between the  
12 centers of gravity of the basic girder and deck (mm); and  $S_w$  = spacing of interior wheels for  
13 dual lane overload vehicle (mm). The factors  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  are in Tables 3 and 4.

14 The first  $R$  factor given in Table 4 is used to account for negative moment near the pier and skew.  
15  $R$  factors for negative moment load distribution and skew can be multiplied together to find a  
16 combined  $R$  factor when needed.

17 The new equations were developed in a manner to ensure that 95% of the predicted load  
18 distribution factors would not be less than those obtained from FEM analysis, i.e. on the safe side.  
19 The predicted distribution factors are on average 114% of the values from the FEM analysis  
20 results, showing that the equations are conservative (predicting higher girder loading than the  
21 FEM). The standard deviation was 9.6 %. The relationship between the load distribution factors  
22 using the developed equations and those using the finite element analyses is shown in Fig. 11.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 The bold line in the figure indicates the expected result if the two analyses matched perfectly.  
2 Most of the data points in the figure are at the upper side of the bold line indicating that the  
3 analysis using the developed equations is conservative (predict larger load distribution factors  
4 than the FEM).

5 A comparison of the load distribution factors for single span bridges subjected to single lane  
6 vehicles calculated from the proposed equations with those from the AASHTO load distribution  
7 factor equations, the finite element analyses (without end diaphragms) and equations by Tabsh  
8 and Tabatabai (2001) was made to investigate and to validate the developed equation. The  
9 AASHTO equation is intended for application with the AASHTO design truck that has a 1829  
10 mm transverse wheel spacing while the single lane overload vehicle had an 2438 mm transverse  
11 wheel spacing. The load distribution factors for the overload vehicle are, therefore, expected to  
12 be less than those calculated from the AASHTO load distribution factor equations because of the  
13 wider wheel spacing. The equations by Tabsh and Tabatabai (2001) were developed to consider  
14 the variation of the lateral wheel spacing of the single lane vehicles.

15 The comparison results are shown in Fig. 12. The new load distribution factors are clearly lower  
16 than would be obtained using the equations directly from AASHTO T4.6.2.2.2b-1 and  
17 T4.6.2.2.3a-1 as expected. Using the AASHTO equations directly would overestimate the  
18 overload vehicle effects by as much as 25%. This may be appropriate for normal truck loading  
19 where the vehicle geometry and weight varies, but may be too conservative for overload vehicles  
20 with well known weights travelling at slow speeds. The load distribution factors calculated from  
21 the proposed equations are approximately 14% higher than more accurate values from the finite  
22 element analyses. The proposed equations are capable of replacing the time consuming 3D finite  
23 element and may still provide a safe or conservative result for unusual overload vehicles on

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 common bridges since bounding load cases were used, the standard deviation from FEM results  
2 was only 9.6%, and 95% of the predicted values were higher than the FEM values.

3 Additional comparison results are shown in Figs. 13 and 14. No end diaphragms were used in the  
4 results shown in the figures.

5 Fig. 13 shows the results for single span concrete I girder bridges with skew under overload  
6 vehicles and Fig. 14 shows the results for negative moment load distribution factors for two span  
7 concrete I girder bridges without skew subjected to overload vehicles. The results shown in Figs.  
8 12, 13 and 14 indicate that the developed equations have a wide range of applicability.

9  
10 **Summary and Conclusions**

11 Three dimensional finite element analysis techniques were used to analyze a large series of  
12 multi-girder bridges under various oversize overload vehicles. 118 multi-girder bridges with 16  
13 load cases of oversize overload vehicles for each bridge were used in the finite element analyses.  
14 The variables in configuration of the bridges included span length, deck depth, girder spacing,  
15 girder type, girder stiffness, skew angle, number of spans, and use of end diaphragms. The  
16 overload vehicle types varied with single lane and dual lane/trailer vehicles, and in the transverse  
17 spacing of interior wheels for dual lane vehicles. Resulting shear and moments in girders were  
18 both examined.

19 Load distribution factor equations for the multi-girder bridges under oversize overload vehicles  
20 were successfully developed based on the FEM analysis results. The equations are for  
21 determining the amount of shear and moment induced in a girder due to the passage of overload  
22 vehicles.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 The result of these analyses and development of the new load distribution equations for the  
2 oversize overweight vehicles led to the following conclusions:

- 3
- 4 1. The finite element analysis technique using shell elements for the deck, frame element for  
5 girders and rigid links to connect the shell element and frame element accurately simulated  
6 an actual bridge load test result within 8 % of the measured deflection.
- 7 2. The load distribution factors decrease as the skew angle increases. This was more evident  
8 for the shear load distribution factor and the dual lane trailer loading case.
- 9 3. End diaphragms serve to reduce the load distribution factors. Analysis of bridges,  
10 neglecting end diaphragms, provides a conservative estimate of the distribution factors.
- 11 4. The positive moment load distribution factors for 2 span bridges are not significantly  
12 different compared with those of single span bridges. The negative moment load  
13 distribution factors of 2 span bridges are considerably different than the positive moment  
14 load distribution factors of the single span bridges. Negative moments should be calculated  
15 using these special factors.
- 16 5. The proposed equations to predict load distribution factors for oversize overweight  
17 vehicles provide a simple fast means of evaluating the forces that special trucks will create  
18 in bridges, especially for the permitting process. The equations provided results on average  
19 114% of the values from the more accurate finite element analysis results. The proposed  
20 equations are conservative (predicting higher girder loading than the FEM). The empirical  
21 equations were found to be capable of replacing the time consuming 3D finite element  
22 analysis with conservative results. The equations can be used for single and continuous  
23 span bridges and bridges with skew.

1  
2  
3  
4 1  
5  
6  
7 2 **References**  
8

- 9 3 AASHTO (2009). "AASHTO LRFD Bridge Design Specifications, 2009 Interims." 4th Ed.,  
10  
11 4 American Association of State Highway and Transportation Officials.  
12  
13  
14 5 Bae, H., and Oliva, M. G. (2010). "Bridge Analysis and Evaluation of Effects under Overload  
15  
16 6 Vehicles." *Rep. No. CFIRE02-03*, Univ. of Wisconsin, Madison, WI, <  
17  
18 7 <http://www.wistrans.org/cfire/Research/CFIRE/GY02/03/index.html>>.  
19  
20  
21 8 Bishara, A. G., Liu, M. C., and El-Ali, N. D. (1993). "Wheel Load Distribution on Simply  
22  
23 9 Supported Skew I-beam Composite Bridges." *J. Struct. Eng.*, ASCE, 119(2), 399-419.  
24  
25  
26 10 Chou, C. J. (1996). "Effect of Overloaded Heavy Vehicles on Pavement and Bridge Design."  
27  
28 11 *Transportation Research Record*, N1539, 58-65.  
29  
30  
31 12 Chou, K. C., Deatherage, J. H., Letherwood, T. D., and Khayat, A. J. (1999). "Innovative  
32  
33 13 Method for Evaluating Overweight Vehicle Permits." *J. Bridge Eng.*, 4(3), 221-227.  
34  
35  
36 14 Conachen, M. (2005). "Modular 3-D FRP Reinforcing System for a Bridge Deck in Fond du Lac,  
37  
38 15 Wisconsin." MS Thesis, Dept. of Civil and Environmental Engineering, Univ. of Wisconsin,  
39  
40 16 Madison, WI.  
41  
42  
43 17 Fu, G., and Hag-Elsafi, O. (1996a). "A Bridge Live Load Model including Overloads."  
44  
45 18 Probabilistic Mechanics and Structural and Geotechnical Reliability, *Proc. of the Seventh*  
46  
47 19 *Specialty Conference*, Worcester, MA, August 7-9, 34-37.  
48  
49  
50 20 Fu, G., and Hag-Elsafi, O. (1996b). "New Safety-Based Checking Procedure for Overloads on  
51  
52 21 Highway Bridges." *Transportation Research Record*, N1541, 22-28.  
53  
54  
55 22 Hays, C. O. (1984). "Evaluating Bridge Overloads using the Finite Element Method." *Official*  
56  
57 23 *Proceedings - International Bridge Conference*, Pittsburgh, PA, USA, 232-238.  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 Tabsh, S. W., and Tabatabai, M. (2001). "Live Load Distribution in Girder Bridges Subject to  
2 Oversized Trucks." *J. Bridge Eng.*, 6(1), 9-16.

3

4



1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 **List of Tables**

2 Table 1. Selected configuration sets for bridges in the finite element analyses

3 Table 2. List of configurations for selected multi-girder bridges with single span without skew  
4 and diaphragms.

5 Table 3. Constants and exponents for the proposed load distribution factor equations for oversize  
6 overload vehicles

7 Table 4. R factor for proposed load distribution factor equations for overload vehicles

8

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1 Table 1. Selected configuration sets for bridges in the finite element analyses

Variables	Parameters
Span (m)	15.2, 24.4, 36.6 and 45.7
Girder spacing (mm)	1524, 2438, 3353 and 4267
Deck depth (mm)	152, 229 and 305
Girder type	steel girder type 1, steel girder type 2, concrete I girder and wide flange concrete girder
Skew (degree)	0, 20, 40, 50 and 60
# of Spans	1 and 2
End diaphragm	with end diaphragms and without end diaphragms

2  
3

1  
2  
3  
4 1 Table 2. List of configurations for selected multi-girder bridges with single span without skew  
5  
6  
7 2 and diaphragms.  
8

Set 1: Variable = deck depth			
Case ID	Span (m)	Girder spacing (mm)	Deck depth (mm)
1	24.4	2438	152
2	24.4	2438	229
3	24.4	2438	305
4	24.4	3353	152
5	24.4	3353	229
6	24.4	3353	305
7	36.6	2438	152
8	36.6	2438	229
9	36.6	2438	305
Set 2: Variable = girder spacing			
Case ID	Span (m)	Girder spacing (mm)	Deck depth (mm)
10	24.4	1524	152
11	24.4	4267	152
12	24.4	1524	229
13	24.4	4267	229
14	36.6	1524	229
15	36.6	3353	229
16	36.6	4267	229
Set 3: Variable = span			
Case ID	Span (m)	Girder spacing (mm)	Deck depth (mm)
17	15.2	2438	152
18	45.7	2438	152
19	15.2	2438	229
20	45.7	2438	229
21	15.2	3353	305
22	36.6	3353	305
23	45.7	3353	305

3

4

1  
2  
3  
4 1 Table 3. Constants and exponents for the proposed load distribution factor equations for oversize  
5  
6  
7 2 overload vehicles  
8

		C	a	b	c	d	e
Single lane trailer	Moment	$8.55 \times 10^{-2}$	0.38	-0.37	-0.20	0.03	
	Shear	$0.34 \times 10^{-2}$	0.62	-0.09	-0.10	0.04	
Dual lane trailer	Moment	$1.72 \times 10^{-2}$	0.47	-0.27	0.03	0.03	-0.10
	Shear	$1.01 \times 10^{-2}$	0.74	-0.12	-0.11	0.04	-0.28

9  
10  
11  
12  
13  
14  
15 3  
16  
17  
18 4  
19  
20 5  
21  
22

23 6 Table 4. R factor for proposed load distribution factor equations for overload vehicles  
24

		R
Negative moment load distribution factor (for single lane and dual lane loading)		1.3
Bridges with Skew* ( $\theta$ = skew angle)	Moment load distribution factor for single lane loading	$1 - 0.05 \tan^2 \theta$
	Shear load distribution factor for single lane loading	$1 - 0.23 \tan \theta$
	Moment load distribution factor for dual lane loading	$1 + 0.19 \tan^2 \theta - 0.55 \tan \theta$
	Shear load distribution factor for dual lane loading	$1 + 0.25 \tan^2 \theta - 0.76 \tan \theta$
All other cases		1.0

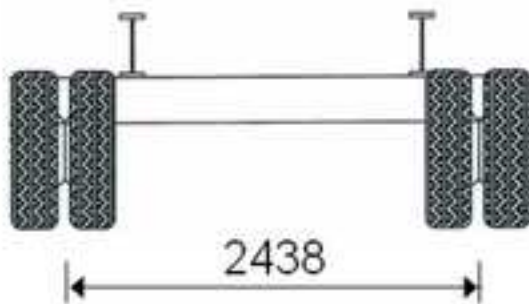
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40 7 \* Valid for  $\theta = 0^\circ \sim 60^\circ$   
41  
42  
43 8  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

**Figure 01**

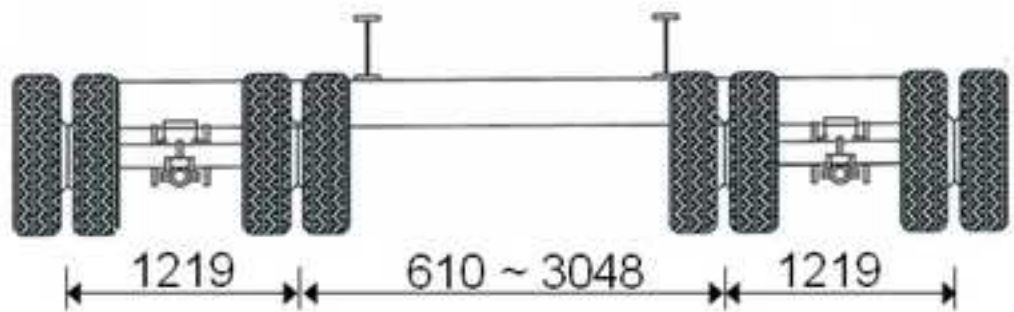
[Click here to download high resolution image](#)



Fig. 1. Special overload vehicle (Photograph courtesy of Perkins Motor Transport)



a) Single lane trailer

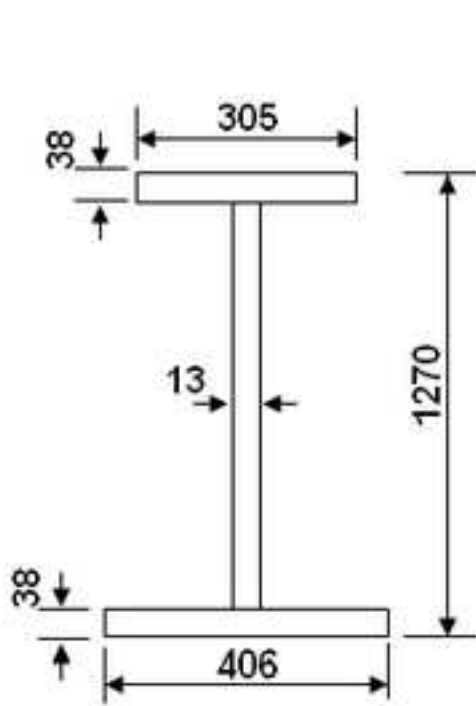


b) Dual lane trailer

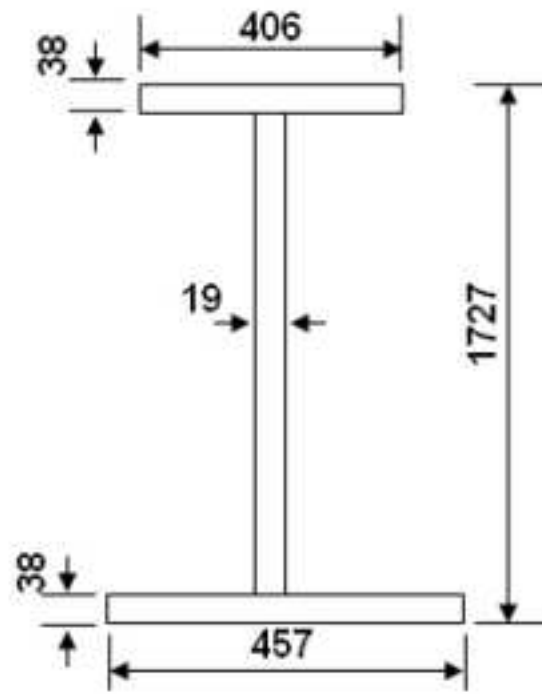
Fig. 2. Transverse wheel spacing of the single lane and dual lane trailers

Figure 03

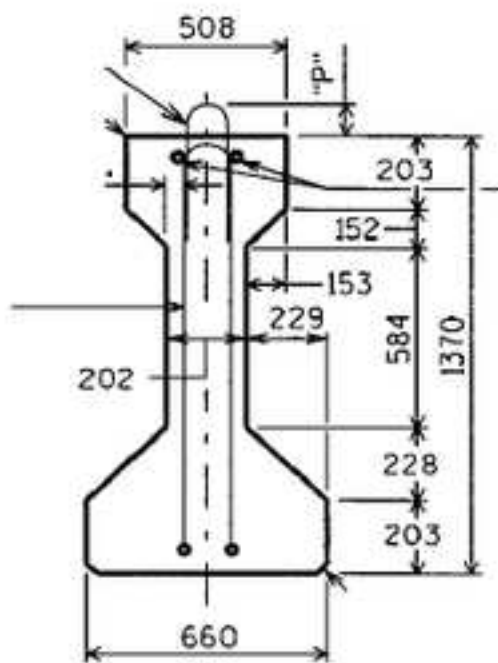
[Click here to download high resolution image](#)



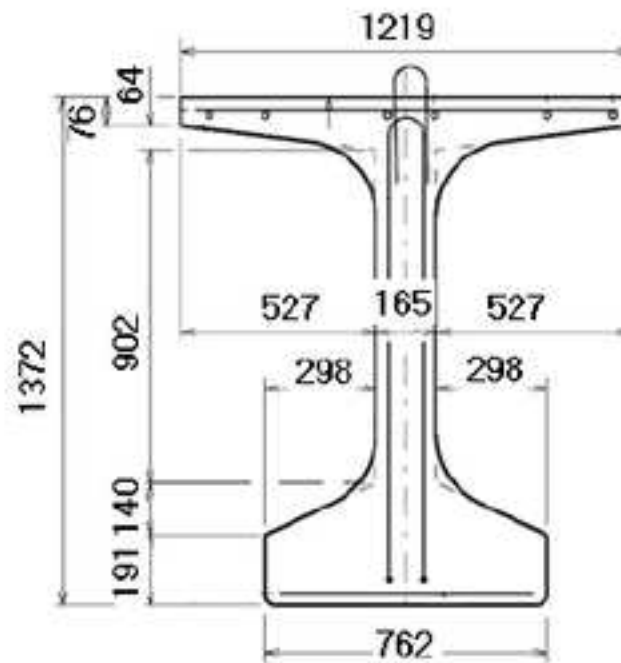
a) Steel girder type 1



b) Steel girder type 2

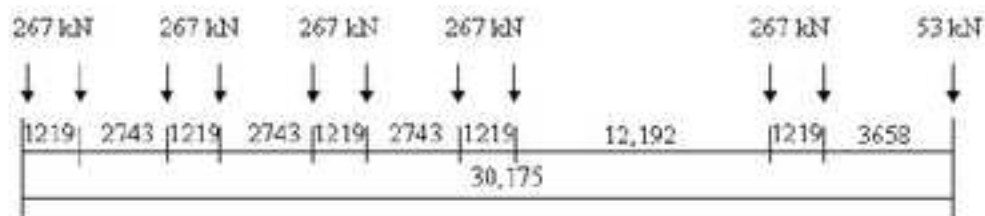


c) Concrete I girder

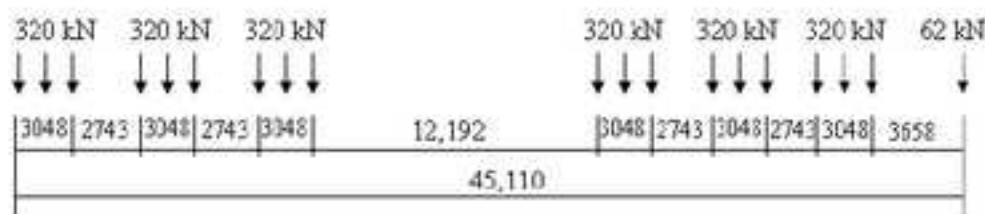


d) Wide flange concrete girder

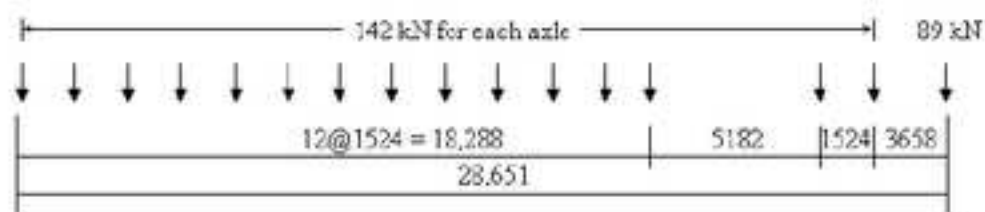
Fig. 3. Selected girder types for multi-girder analysis



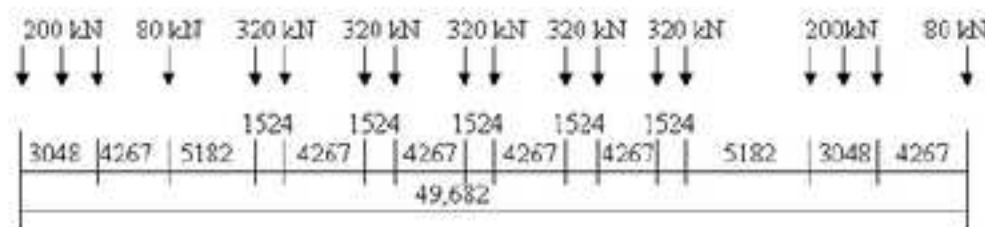
a) Single lane loading case 01 (Gross weight = 1388 kN)



b) Single lane loading case 02 (Gross weight = 1982 kN)



c) Dual lane loading case 01 (Gross weight = 2219 kN)



d) Dual lane loading case 02 (Gross weight = 2160 kN)

Fig. 4. Axle spacings and loads of selected overload vehicles for analysis



Figure 05

[Click here to download high resolution image](#)

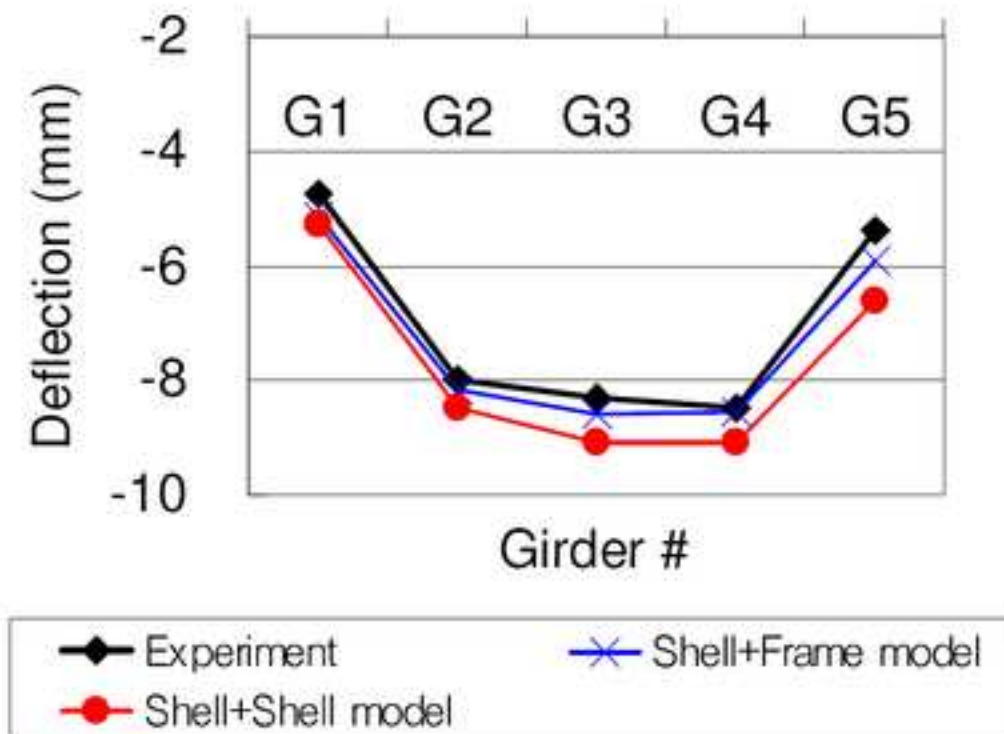
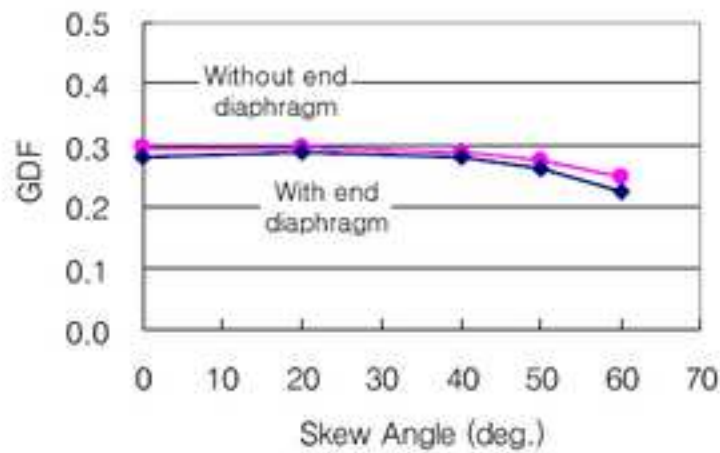


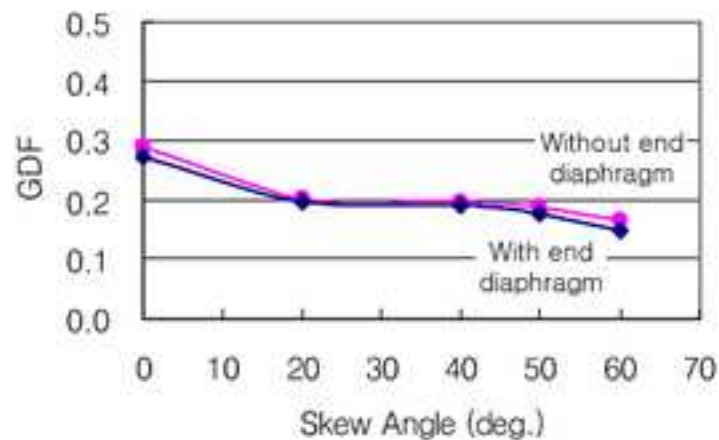
Fig. 5. Cross section deflection prediction results for verification of FEM analysis technique

Figure 06

[Click here to download high resolution image](#)



a) Single lane trailer ( $L = 36.6\text{m}$ ,  $S = 2438\text{ mm}$ , and  $t = 229\text{ mm}$ )



b) Dual lane trailer ( $L = 36.6\text{m}$ ,  $S = 2438\text{ mm}$ ,  $t = 229\text{ mm}$ , and  $S_w = 1829\text{ mm}$ )

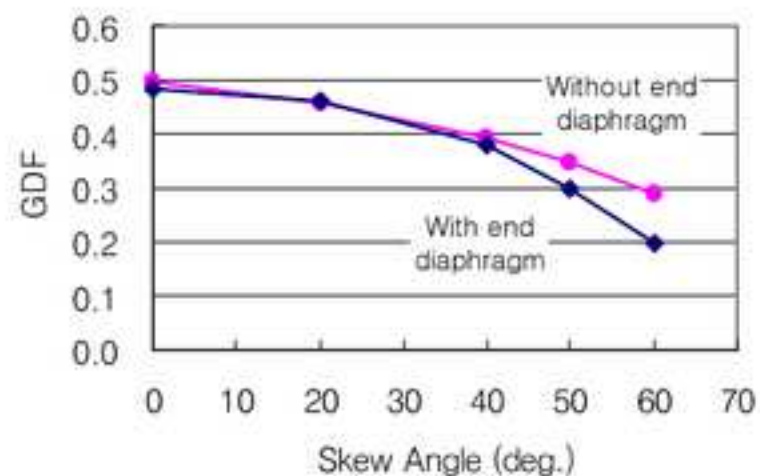
Fig. 6. Moment girder load distribution factors (GDF) of single span concrete I girder bridges

with skew and oversize overload vehicle (Case-8 bridge configurations in Table 2)

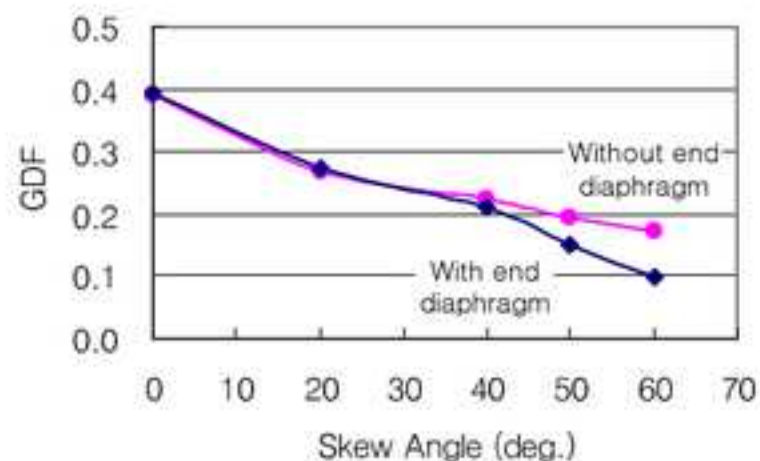
( $L$  = span length,  $S$  = spacing of girders,  $t$  = deck depth and  $S_w$  = transverse spacing of center wheels)

Figure 07

[Click here to download high resolution image](#)



a) Single lane trailer ( $L = 36.6\text{m}$ ,  $S = 2438\text{ mm}$ , and  $t = 229\text{ mm}$ )

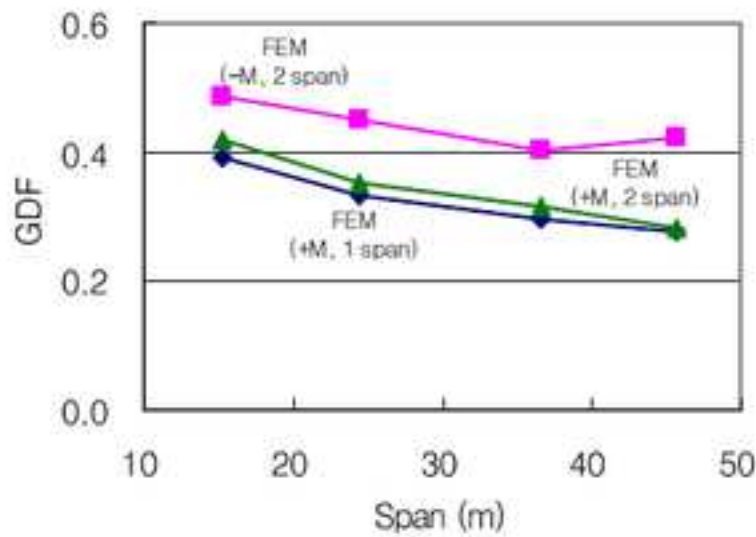


b) Dual lane trailer ( $L = 36.6\text{m}$ ,  $S = 2438\text{ mm}$ ,  $t = 229\text{ mm}$ , and  $S_w = 1829\text{ mm}$ )

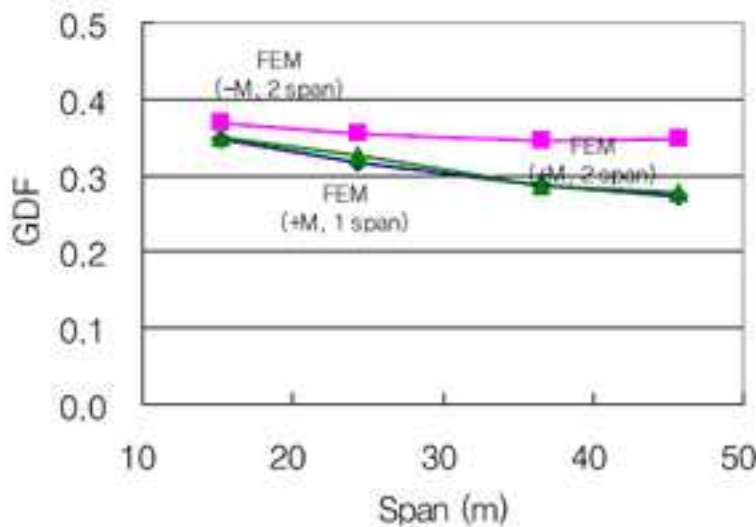
Fig. 7. Shear girder load distribution factors (GDF) of single span concrete I girder bridges with skew and oversize overload vehicle (Case-8 bridge configurations in Table 2)

Figure 08

[Click here to download high resolution image](#)



a) Single lane trailer ( $S = 2438$  mm and  $t = 229$  mm)

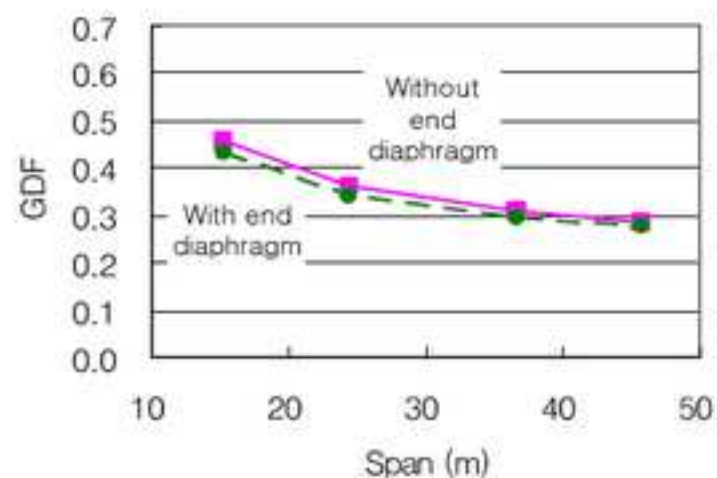


b) Dual lane trailer ( $S = 2438$  mm,  $t = 229$  mm, and  $S_w = 1829$  mm)

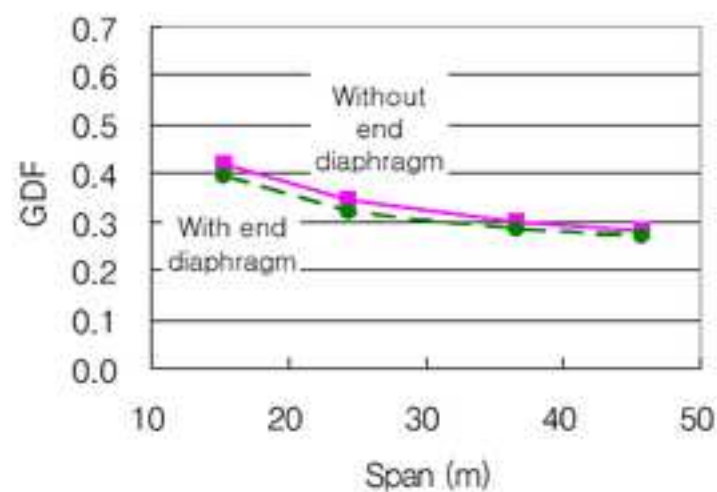
Fig. 8. Positive and negative moment girder load distribution factors (GDF) for two span concrete I girder bridges and positive moment girder distribution factors for single span bridges, all without skew or end diaphragms and under oversize overload vehicles

**Figure 09**

[Click here to download high resolution image](#)



a) Single lane trailer ( $S = 3353$  mm and  $t = 305$  mm)

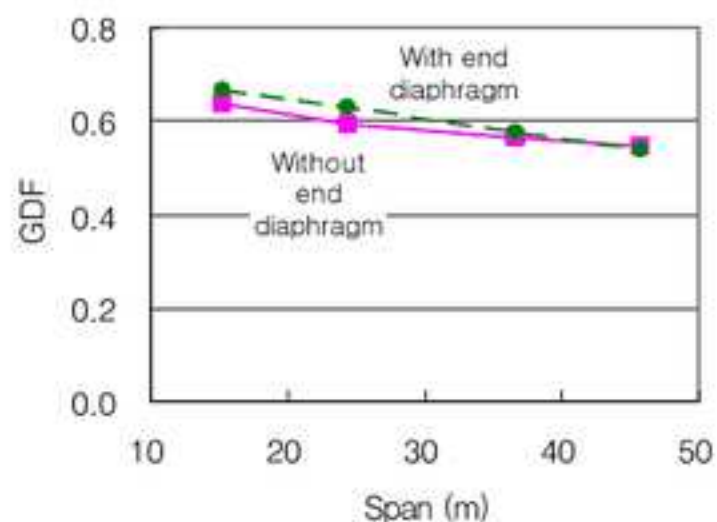


b) Dual lane trailer ( $S = 3353$  mm,  $t = 305$  mm, and  $S_w = 1829$  mm)

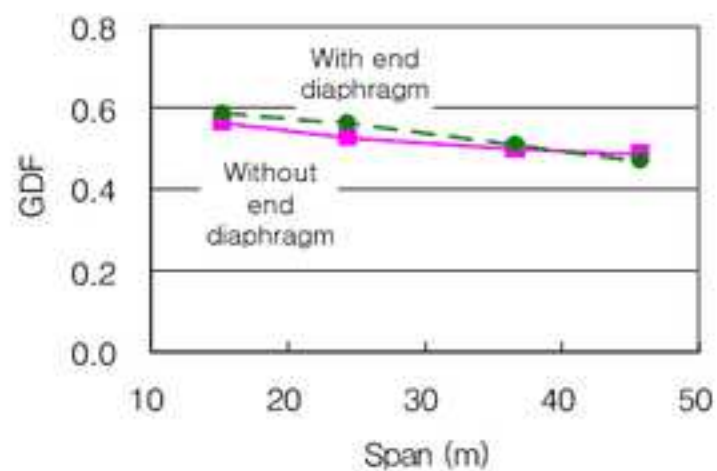
Fig. 9. Moment girder load distribution factors (GDF) comparison of cases with end diaphragms and those without end diaphragms for single span concrete I girder bridges without skew

Figure 10

[Click here to download high resolution image](#)



a) Single lane trailer ( $S = 3353$  mm and  $t = 305$  mm)



b) Dual lane trailer ( $S = 3353$  mm,  $t = 305$  mm, and  $S_w = 1829$  mm)

Fig. 10. Shear girder load distribution factors (GDF) comparison of cases with end diaphragms and those without end diaphragms for single span concrete I girder bridges without skew

**Figure 11**

[Click here to download high resolution image](#)

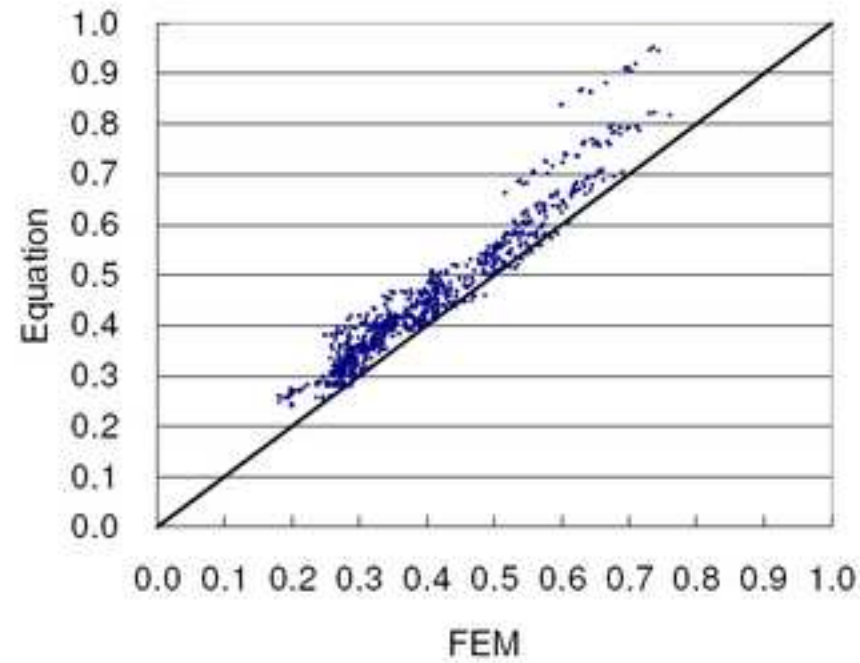
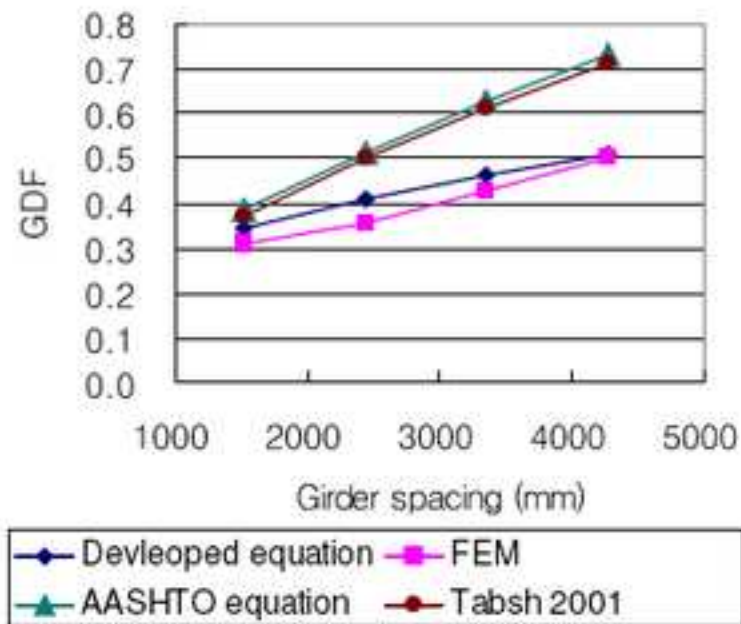


Fig. 11. Comparison of load distribution factors calculated from the developed equation with the finite element analysis results

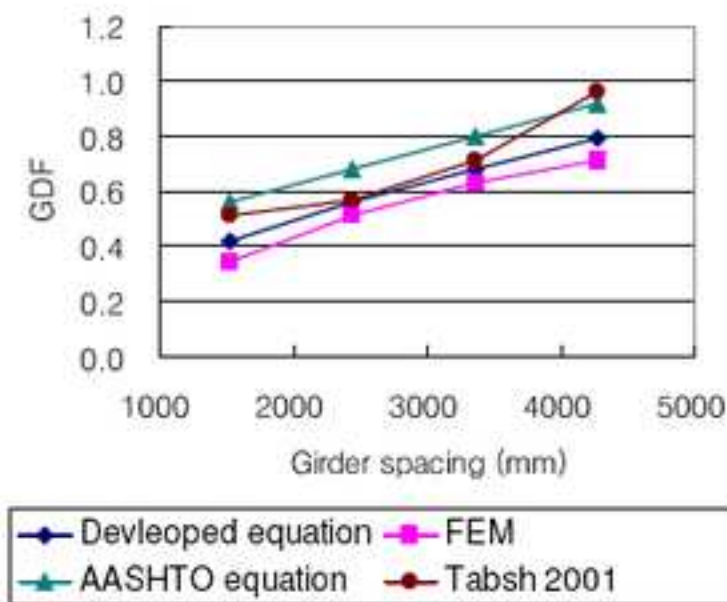


Figure 12

[Click here to download high resolution image](#)



a) Moment GDF ( $L = 24.4$  m and  $t = 152$  mm)



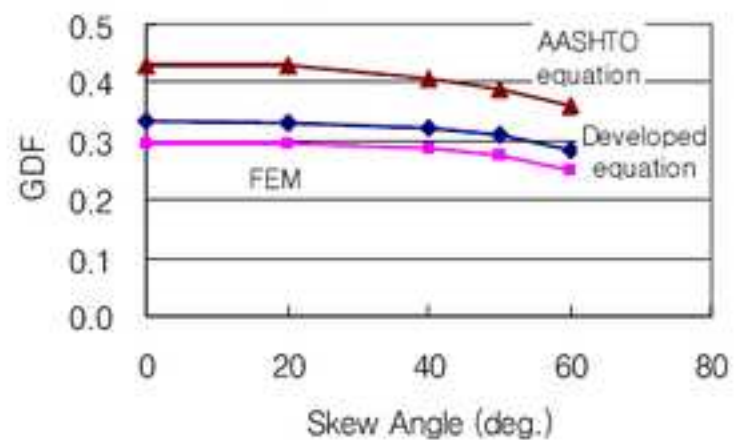
b) Shear GDF ( $L = 24.4$  m and  $t = 152$  mm)

Fig. 12. Girder load distribution factors (GDF) for single span steel girder bridges without skew under the single lane vehicle using AASHTO equations, equations by Tabsh and Tabatabai (2001), proposed equations and finite element analysis

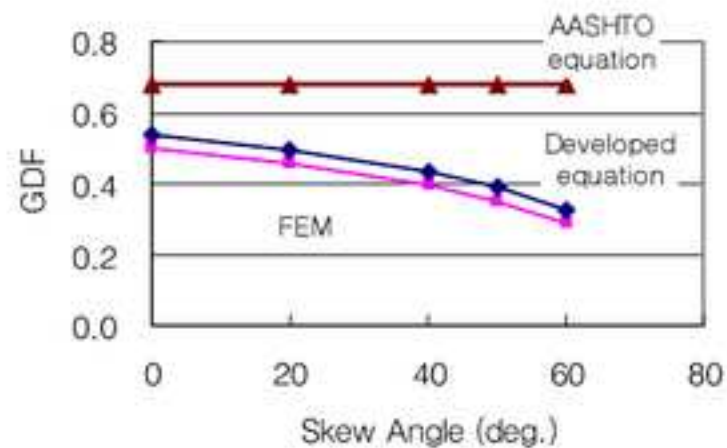


**Figure 13**

[Click here to download high resolution image](#)



a) Moment GDF ( $L = 36.6$  m,  $S = 2438$  mm, and  $t = 229$  mm)

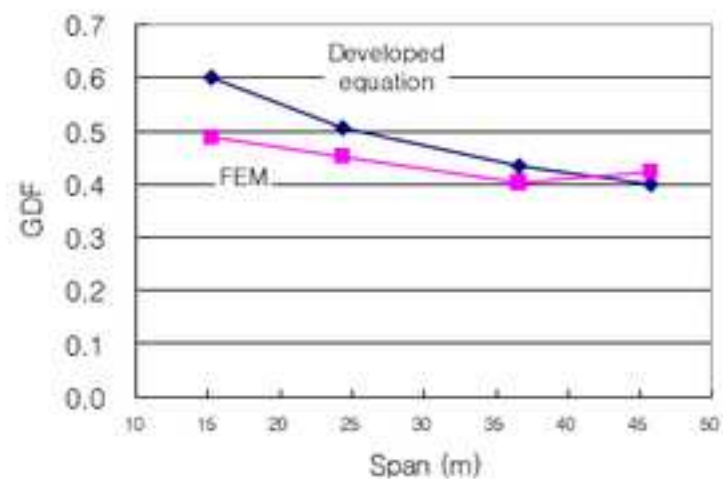


b) Shear GDF ( $L = 36.6$  m,  $S = 2438$  mm, and  $t = 229$  mm)

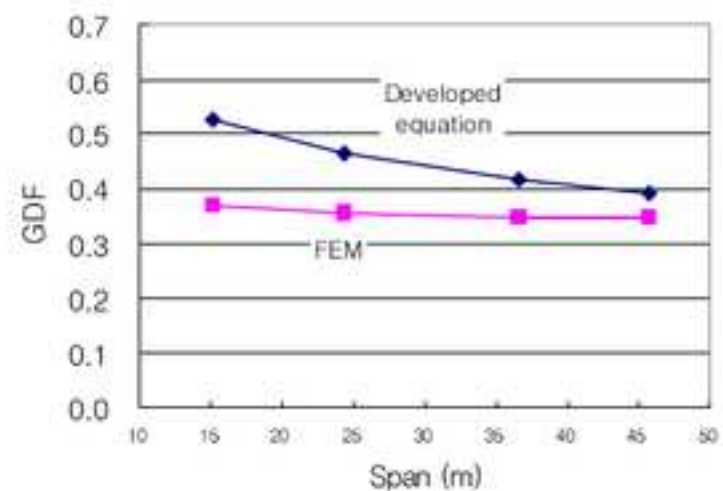
Fig.13. Girder load distribution factors (GDF) for single span concrete I girder bridges with skew under single lane oversize overload vehicle using AASHTO equations, proposed equation and finite element analysis

**Figure 14**

[Click here to download high resolution image](#)



a) Single lane trailer ( $S = 2438$  mm and  $t = 229$  mm)



b) Dual lane trailer ( $S = 2438$  mm,  $t = 229$  mm and  $S_w = 1829$  mm)

Fig. 14. Negative moment girder load distribution factors (GDF) for 2 span concrete I girder bridges without skew under overload vehicle using proposed equation and finite element analysis

## List of Figures

Fig. 1. Special overload vehicle (Photograph courtesy of Perkins Motor Transport)

Fig. 2. Transverse wheel spacing of the single lane and dual lane trailers

Fig. 3. Selected girder types for multi-girder analysis

Fig. 4. Axle spacings and loads of selected overload vehicles for analysis

Fig. 5. Cross section deflection prediction results for verification of FEM analysis technique

Fig. 6. Moment girder load distribution factors (GDF) of single span concrete I girder bridges with skew and oversize overload vehicle (Case-8 bridge configurations in Table 2)

( $L$  = span length,  $S$  = spacing of girders,  $t$  = deck depth and  $S_w$  = transverse spacing of center wheels)

Fig. 7. Shear girder load distribution factors (GDF) of single span concrete I girder bridges with skew and oversize overload vehicle (Case-8 bridge configurations in Table 2)

Fig. 8. Positive and negative moment girder load distribution factors (GDF) for two span concrete I girder bridges and positive moment girder distribution factors for single span bridges, all without skew or end diaphragms and under oversize overload vehicles

Fig. 9. Moment girder load distribution factors (GDF) comparison of cases with end diaphragms and those without end diaphragms for single span concrete I girder bridges without skew

Fig. 10. Shear girder load distribution factors (GDF) comparison of cases with end diaphragms and those without end diaphragms for single span concrete I girder bridges without skew

Fig. 11. Comparison of load distribution factors calculated from the developed equation with the finite element analysis results

Fig. 12. Girder load distribution factors (GDF) for single span steel girder bridges without skew under the single lane vehicle using AASHTO equations, equations by Tabsh and Tabatabai (2001), proposed equations and finite element analysis

Fig.13. Girder load distribution factors (GDF) for single span concrete I girder bridges with skew under single lane oversize overload vehicle using AASHTO equations, proposed equation and finite element analysis

Fig. 14. Negative moment girder load distribution factors (GDF) for 2 span concrete I girder bridges without skew under overload vehicle using proposed equation and finite element analysis



COPYRIGHT TRANSFER AGREEMENT

Manuscript Number: BEENG-573

Type: Technical Paper

Publication Title: Moment and Shear Load Distribution Factors for Multi-girder Bridge Subjected to Overloads

Manuscript Authors: Han Ug Bae and Michael G. Oliva

Corresponding Author Name and Address: Han Ug Bae, Dept. of Civil and Environmental Eng. Univ. of Wisconsin, Madison, 1415 Engineer Dr., Madison, WI 53706

This form must\* be returned with your final manuscript to: American Society of Civil Engineers, Journals Production Services Dept., 1801 Alexander Bell Drive, Reston, VA 20191-4400.

The author(s) warrant(s) that the above cited manuscript is the original work of the author(s) and has never been published in its present form.

The undersigned, with the consent of all authors, hereby transfers, to the extent that there is copyright to be transferred, the exclusive copyright interest in the above-cited manuscript (subsequently called the "work"), in this and all subsequent editions of this work, and in derivatives, translations, or ancillaries, in English and in foreign translations, in all formats and media of expression now known or later developed, including electronic, to the American Society of Civil Engineers subject to the following.

• The undersigned author and all coauthors retain the right to revise, adapt, prepare derivative works, present orally, or distribute the work provided that all such use is for the personal noncommercial benefit of the author(s) and is consistent with any prior contractual agreement between the undersigned and/or coauthors and their employer(s).

• In all instances where the work is prepared as a "work made for hire" for an employer, the employer(s) of the author(s) retain(s) the right to revise, adapt, prepare derivative works, publish, reprint, reproduce, and distribute the work provided that such use is for the promotion of its business enterprise and does not imply the endorsement of ASCE.

• No proprietary right other than copyright is claimed by ASCE.

• An author who is a U.S. Government employee and prepared the above-cited work does not own copyright in it.

If at least one of the authors is not in this category, that author should sign below. If all the authors are in this category, check here  and sign here: \_\_\_\_\_ . Please return this form by mail.

SIGN HERE FOR COPYRIGHT TRANSFER [Individual Author or Employer's Authorized Agent (work made for hire)]

Print Author's Name: Han Ug Bae Signature of Author (in ink): Han Ug Bae

Print Agent's Name and Title: Signature of Agency Rep (in ink):

Date: 01/30/2011

Note: If the manuscript is not accepted by ASCE or is withdrawn prior to acceptance by ASCE, this transfer will be null and void and the form will be returned to the author.

\*Failure to return this form will result in the manuscript's not being published.

# ASCE Worksheet for Sizing Technical Papers & Notes

**\*\*\*Please complete and save this form then email it with each manuscript submission.\*\*\***

**Note:** The worksheet is designed to automatically calculate the total number of printed pages when published in ASCE tw format.

<b>Journal Name:</b>	<b>tribution Factors for Multi-girder Br</b>	<b>Manuscript # (if known):</b>	<b>BEENG-57</b>
<b>Author Full Name:</b>	<b>Han Ug Bae and Michael G. Oliva</b>	<b>Author Email:</b>	<b>bae@wisc.edu, hanug01</b>

The maximum length of a technical paper is 10,000 words and word-equivalents or 8 printed pages. A technical note should not exceed 3,500 word-equivalents in length or 4 printed pages. Approximate the length by using the form below to calculate the total number of words in the text to the total number of word-equivalents of the figures and tables to obtain a grand total of words for the paper/note to fit ASCE format. Over must be approved by the editor; however, valuable overlength contributions are not intended to be discouraged by this procedure.

### 1. Estimating Length of Text

**A. Fill in** the four numbers (highlighted in green) in the column to the right to obtain the total length of text.

**NOTE: Equations take up a lot of space.** Most computer programs don't count the amount of space around display equations. Plan on counting 3 lines of text for every simple equation (single line) and 5 lines for every complicated equation (numerator and denominator).

Estimating Length of Text	
Count # of words in 3 lines of text:	43
Divided by 3	3
Average # of words per line	14
Count # of text lines per page	23
# of words per page	329.67
Count # of pages (don't add references & abstract)	12
Title & Abstract	500
Total # refs	10
Length of Text is	4698
	332
	5030
	4

### 2. Estimating Length of Tables

**A. First count** the longest line in each column across adding two characters between each column and one character between each word to obtain total characters.

1-column table = up to 60 characters wide	2-column table = 61 to 120 characters wide
---	--

**B. Then count** the number of text lines (include footnote & titles)

<b>1-column table = up to 60 characters wide by:</b> 17 lines (or less) = 158 word equiv. up to 34 lines = 315 word equiv. up to 51 lines = 473 word equiv. up to 68 text lines = 630 word equiv.	<b>2-column table = 61 to 120 characters wide by:</b> 17 lines (or less) = 315 word equiv. up to 34 lines = 630 word equiv. up to 51 lines = 945 word equiv. up to 68 text lines = 1260 word equiv.
---	---

**C. Total Characters wide by Total Text lines = word equiv.** as shown in the table above. **Add word equivalents** for each table in the column labeled "Word Equivalents."

### 3. Estimating Length of Figures

**A. First reduce** the figures to final size for publication.

**Figure type size can't be smaller than 6 point (2mm).**

**B. Use ruler** and measure figure to fit 1 or 2 column wide format.

1-column fig. = up to 3.5 in.(88.9mm)	2-col. fig. = 3.5 to 7 in.(88.9 to 177.8 mm) wide
---------------------------------------	---

**C. Then use** a ruler to check the height of each figure (including title & caption).

<b>1-column fig. = up to 3.5 in.(88.9mm) wide by:</b> up to 2.5 in.(63.5mm) high = 158 word equiv. up to 5 in.(127mm) high = 315 word equiv. up to 7 in.(177.8mm) high = 473 word equiv. up to 9 in.(228.6mm) high = 630 word equiv.	<b>2-column fig. = 3.5 to 7 in.(88.9 to 177.8 mm) wide by:</b> up to 2.5 in.(63.5mm) high = 315 word equiv. up to 5 in.(127mm) high = 630 word equiv. up to 7 in.(177.8mm) high = 945 word equiv. up to 9 in.(228.6mm) high = 1260 word equiv.
--	--

**D. Total Characters wide by Total Text lines = word equiv.** as shown in the table above. **Add word equivalents** for each table in the column labeled "Word Equivalents."

Total Tables/Figures:	4885
Total Words of Text:	5030

(word equivalents)

Estimating Length of Tables & Figures		
Tables	Word Equivalents	Figures
Table 1	315	Figure 1
2	315	2
3	158	3
4	315	4
5	0	5
6	0	6
7	0	7
8	0	8
9	0	9
10	0	10
11	0	11
12	0	12
13	0	13
14	0	14
15	0	15
		16
		17
		18
		19
		20 and 21

Please double-up tables/figures if additional space is needed (ex. 20+21).

<b>Total words and word equivalents:</b>	<b>9915</b>
<b>printed pages:</b>	<b>8</b>

---

---

two-column

3
16@gmail.c

words and  
t and adding  
length papers

subtototal  
plus headings  
TOTAL words  
printed pages

<b>Figures:</b>
Word Equivalents
158
315
158
315
158
315
315
315
315
315
315
158
315
315
315
0
0
0
0
0
0

## **Response to reviewers' comments**

### **Reviewer: 1**

**Reviewer #1: This manuscript discusses live load distribution factors for over-load vehicles. Technical merit of the research is insignificant. The work does not include in-depth thoughts, but a number of repetitive modeling results are merely presented. The proposed equations are not adequate (the "AASHTO equation" should not have been included- details are described below). Nevertheless, the idea of the present research is still valuable for the bridge community. The reviewer recommends i) the format of the proposed equations be revised to exclude the AASHTO Eq. and ii) a reliability analysis be conducted to examine the safety margin of the proposed equation, and resubmit a revised manuscript. Otherwise, this manuscript is not acceptable.**

Thank you for your considerate review. The reviewer is correct in noting that the approach taken was not an in-depth academic study of how load distribution occurs between girders in bridges, but rather a focused study on simple and practical means of estimating the distribution for common types of bridges such as grade crossings under very large overload vehicles. The study was initiated at the request of a state transportation department that has to deal with permit decisions in those situations. The authors have tried to clarify the intent of the manuscript in the introduction based on the reviewer's comments. (Page 2, Line 18) Please refer to other detailed responses listed below.

#### **Page 2 line 10- provide reference after '356 kN'**

The reference was provided. (Page 2, Line 10)

#### **Page 4 line 19- provide reference after 'largest' and 'effect'**

References were provided. (Page 5, Line 4 and Page 5, Line 5)

In the first reference after 'largest' (Tabsh and Tabatabai 2001), it is stated that "the first interior girder was found to be the most critical interior girder in both flexure and shear.

In the second reference after 'effect' (Bishara et al. 1993), bridges with 5~8 girders were analyzed to find load distribution factors and found that bridges with 5 girders had the largest load distribution factors.

#### **Page 5 line 13- add reference after 'data'**

The data was not found from any reference. It was collected from major transporters in United States by the authors. This is clearly stated now. (Page 5, Line 22)



**Page 6 line 2- provide brief information on the Wisconsin bridge and delete 'the University of ...Wisconsin'**

'the University of ...Wisconsin' was deleted. (Page 6, Line 10)

The brief information on the bridge was provided as “The superstructure of the bridge consists of two continuous spans having a length of 32.81 m. The cross section consists of five prestressed concrete girders equally spaced, supporting a reinforced concrete deck. The bridge was subjected to 2 lane truck loading at the mid-span of the first span and deflections of the five girders were measured at the mid-span of the first span.” (Page 6, Line 11)

**Page 6 line 4- why two different programs were used?**

Analysis results and description related to ABAQUS were removed from this manuscript. (Comments from Reviewers # 1 and #2)

**Page 6 line 12- ABAQUS was not used for the main analysis. Please delete it from page 6 line 4 and Fig. 5**

Analysis results and description related to ABAQUS were removed from this manuscript. (Comments from Reviewers # 1 and #2)

**Page 6 line 19- '27.6 and 55.1' why such strengths were used?**

They are the values generally used in United States. (Page 7, Line 7) This is now stated in the text.

**Page 6 line 22- 'transverse location' not clear why transverse locations of the vehicles influence the maximum moment or shear, given only the first interior girder was examined in this research**

The transverse locations of the vehicles inducing maximum member force in the first interior girder are a function of girder spacing and lateral spacing of the vehicle wheels. The locations were found by moving the vehicles in transversal direction. (Page 7, Line 11)

**Page 7 line 15- explain why such results were obtained. The concept of basic mechanics may be of help.**

A reference to Bishar's work has been added to the text. (Page 8, Line 7) It is well explained by Bishara et al. (1993). In the reference, it is explained as “It can be concluded that skew angles always reduce the distribution factor. This might be due to the fact that some of the wheels of

trucks on skew bridges are closer to the supports than on right bridges. Another reason may be that in short spans with large skew angle bridges, the slab tends to bend along a direction perpendicular to the abutments. This action can transfer the load from deck slabs directly to the supports, rather than through the girders as in right bridges.”

**Page 9 line18 (Eqs. 1 and 2)- the present format is not acceptable. AASHTO equations already include all the variables shown in Eqs. 1 and 2. This is not a recommendable approach. The author performed curve-fitting, which means that empirical equations without considering the AASHTO eq could have been developed. Furthermore, the AASHTO eq includes a multiple presence factor that is not accounted for in the present research. Significant technical discrepancy exists.**

New equations without considering the AASHTO eq. were developed and shown in Eqs. (1)~(2) and Table 3 and 4.

**Page 10 line 8-9- this is what the reviewer meant in page 9 line 18. No need to duplicate the terms and the present approach is not adequate.**

New equations without considering the AASHTO eq. were developed and shown in Eqs. (1)~(2) and Table 3 and 4.

**Page 11 line 10- this is not reasonable. The code equation should be always be conservative due to safety. The code equations estimate distribution factors reasonably well here. What would be the benefit of proposing a new eq even though the existing eq shows good agreement? Insignificant improvement of the prediction is worthwhile to spend significant time and efforts?**

The authors agree that code methods should be always conservative due to a need for safety. The code force distributions equations were developed based on standard trucks which have different lateral wheel spacings than the typical overload truck and engineers should thus be hesitant to use those equations. The existing equations may also be too conservative to apply to the oversize overweight vehicles. It seems to be reasonable to develop equations based on the actual configurations of the vehicles. The proposed equations are also conservative as shown in Fig. 11. We are not suggesting that the new equations must be used, only that they are less in error than the existing code equations based on normal trucks.

It is philosophically questionable where the correct position to introduce safety factors exists. Our codes have typically not applied safety factors to all input and steps in an analysis. In this case is it appropriate for safety factors to be applied to the service overload and then additional safety factors for how that overload is transferred to girders? Our approach is to assume that safety factors should

be lumped at key steps, such as in selecting the load, and that remaining steps should provide accurate representation of the bridge behavior.

**Page 11 line 13- 'as much as 25%' this is a very good predictive margin**

This margin seems to be good for regular vehicles. However, this seems to be too conservative for the overloads. The weight of overload items and the overload truck is well controlled and a safety margin is provided through code load factors. A alternative approach might be to place safety factors on items such as the girder distribution factors and then change the code load factors for special overload trucks with known weight that are travelling at slow speeds. Discussion to this effect has been added to the text. (Page 11, Line 18)

**Page 11 line 16- not sure how the authors can claim this sentence without performing a safety analysis. A reliability analysis needs to be conducted.**

Unfortunately a complete reliability analysis is beyond the scope of the present research. The research was simply intended to provide a Wisconsin transportation authority with a simple and practical means of judging the likely forces that overload vehicles could place on girders. The proposed equations are judged to be conservative due to the following reasons. Added discussion has been placed in the text. (Page 11, Line 22)

- 1) The equations were developed based on analysis of 118 bridges and 16 bounding load cases for each bridge. This covers most of the possible bridge configuration and bounds most load cases.
- 2) The predicted distribution factors using the proposed equations are on average 114% of the values from the FEM analysis results and the standard deviation was 9.6 %.
- 3) 95% of the predicted values using the developed equations were higher than the values from FEM analysis.
- 4) Additional safety margins are provided through load factors.

## Reviewer # 2

**Overall this paper presents an interesting topic that is appropriate for the Journal of Bridge Engineering, but the manner in which it is presented needs significant improvement before the manuscript is published. To highlight the major shortcomings of the manuscript, the work performed basically represents a parametric study on load distribution behavior (moment and shear) for multi-girder bridges subjected to very large overload vehicles which are beyond the scope of overload studies; however the insufficient depth is provided to describe the investigation performed (details are lacking). In this reviewer's opinion, this lack of depth makes the manuscript unpublishable in its current form. The following comments are intended to aid the authors in their revision.**

Thank you for your considerate review of this paper. The authors have tried to revise the paper to provide sufficient depth based on the reviewer's comments.

**Pg 2 – line 21 – The authors describe the vehicles used in the AASHTO LRFD development as standard vehicles, when in fact the vehicles are what would be classified as notional design vehicles that are intended to be representative of the various classes of highway vehicles.**

We have added text making it clear that the standard vehicle is in fact a means of representing the effect of a range of actual normal vehicles. The “standard vehicle” was replaced with the “design truck” as specified in the AASHTO LRFD bridge design manual 2009. (Page 2, Line 22)

**Pg 2 – line 22 – Authors state that standard methods are not applicable, but in reality these methods were not verified for vehicles of the size described in this manuscript. This is somewhat of an odd statement considering that the authors use the AASHTO equations with modification factors later in the manuscript.**

Thank you for pointing this out. We intended to say that there is uncertainty and doubt amongst highway authorities as to whether the methods developed for normal trucks are applicable. We have modified the text to reflect this. (Page 2, Line 24)

New equations without considering the AASHTO eq. were developed and shown in Eqs. (1)~(2) and Table 3 and 4.

**Pg 3 – line 16 – kips should be converted to kN for ASCE consistency.**

400 kips was replaced with 1800 kN. (Page 3, Line 18)

**Pg 3 – line 17 – “...usually limited to specific types of vehicles or bridges.” More details are needed on the types of vehicles and bridges of previous studies and how this study differs.**

It was replaced with “the focus was also usually limited to the single lane trailer or simply

supported bridges without skew.” (Page 3, Line 18)

**Pg 3 – line 19 – “single lane trailers and dual lane trailers,...”, at this point the reader does not know what these are and they should be described and/or highlighted in a figure.**

They are highlighted in a figure. “(Fig. 2)” was added after the “single lane trailers and dual lane trailers” to clearly inform the reader of the type of vehicle. (Page 3, Line 21)

**Pg 3 – line 21 – consider replacing multi girder with multi-girder and 3 dimensional with 3-dimensional**

They were replaced. (Page 3, Line 23)

**Pg 3 – line 21 – replace analysis with analyses**

It was replaced. (Page 4, Line 1)

**Pg 4 – line 3 – the authors present the parameters under investigation, but not the range under consideration. This should be presented in this section, not later (maybe in a table)**

Fig. 2 and Table 1 were added to show the range under consideration. (Page 4, Line 6)

**Pg 4 – line 8 – the authors considered end diaphragms, but no discussion was provided on intermediate bracing which has been shown to have some influence on lateral load distribution behavior (see *Influence of secondary elements and deck cracking on the lateral load distribution of steel girder bridges*. in *Journal of Bridge Engineering* 2006; 11: 178-87). The review would expect for the end diaphragms to have an influence on the shear distribution behavior whereas the intermediate bracing would impact moment distribution factors.**

In AASHTO LRFD bridge design manual (C4.6.2.2.2b), it is commented that the load distribution equations specified in the manual are developed based on the analyses of the bridges without the interior diaphragms within the span since the analyses without the interior diaphragms within the span are conservative. The analyses performed in this research also did not include the interior diaphragms within the span for the same reason.

“The prototype bridges had no intermediate diaphragms within the span since analyses of bridges without the intermediate diaphragms within the span are conservative. (AASHTO 2009-C4.6.2.2.2b)” was added to clarify this. (Page 4, Line 13)

**Pg 4 – line 13 – Five girders were selected for the investigation with no variation in girder spacing which is known to be the primary factor influencing load distribution behavior, this should be justified or explained as to why this choice was made. Also AASHTO LRFD is valid**

**for bridges with 4 girders, why was this not used as the baseline for analyses.**

The line referred to explains that the girder spacing in a single bridge is identical. The spacings of the girder for different bridges considered in the analyses were 1524 mm, 2438 mm, 3353 mm and 4267 mm, as shown in Table 1. Hopefully the addition of table 1 clarifies this.

The bridge data (over 3500 bridges) in United States were analyzed and found that the multi-girder bridges with four girders are approximately 2.9 % of total multi-girder bridges. There is also small chance for the overloads to cross such a narrow bridge with 4 girders. It may not be desirable if the equations become too conservative in order to consider the rare scenario. Therefore, the bridges with four girders were excluded in developing the proposed equations to prevent the equations from being too conservative. This has now been noted in the text. (Page 4, Line 19)

**Pg 4 – line 22 – “... and they are highly dependent on the length of the roadway overhang.” This sentence may also be supplemented because the position of the overload vehicle can also be controlled during crossing to minimize the impact on exterior girders**

The sentence was supplemented. (Page 5, Line 7)

**Pg 6 – line 2 – replace “done” with “performed”**

It was replaced. (Page 6, Line 10)

**Pg 6 – line3 – Conachen 2005 study was on a modular FRP bridge deck?, there has been work done on the distribution behavior of FRP bridges, but this is not the best scenario for verification of the modeling approach for a concrete deck. In addition, whether this is the case or not, more details on the model verification is needed. Simply stating that the model was verified with these field test results is not sufficient (details of the bridge, test configuration, measurement type and location, etc.). This should be summarized such that the reader does not have to retrieve the thesis to get this information. This is a major issue with this manuscript.**

The Conachen 2005 study looked at concrete decks with two types of internal reinforcing material, steel and FRP. Since the deck behavior during the load tests was well within the elastic range it is assumed that the type of reinforcing had little effect on the deck behavior. The test result from the conventional steel reinforced concrete deck was used for the verification and it this is now clarified in the paper. More details on the model verification were also provided as follows.

“The superstructure of the bridge consists of two continuous spans having a length of 32.81 m. The cross section consists of five prestressed concrete girders equally spaced, supporting a reinforced concrete deck. The bridge was subjected to 2 lane truck loading at the mid-span of the first span and deflections of the five girders were measured at the mid-span of the first span.” (Page 6, Line

11)

**Pg 6 – line 4 – The reviewer does not understand why two different analysis programs were used. There are numerous papers available (see *Live Load Distribution Factor for Highway Bridges Based on AASHTOLRFD and Finite Element Analysis in Structures Congress 2006: Structural Engineering and Public Safety - Proceedings of the 2006 Structures Congress* and *Live- Load Distribution Factors for Prestressed Concrete, Spread Box- Girder Bridge in Journal of Bridge Engineering: Volume 11, Issue 5, pp. 573-581 (September/October 2006)* and *Distribution Factors for Curved Continuous Composite Box- Girder Bridges in Journal of Bridge Engineering: Volume 10, Issue 6, pp. 678-692 (November/December 2005)*) that use these programs and the presentation of the two only to use one in the end is unnecessary and adds not value to the manuscript. The modeling approaches are interesting, but have also been successfully employed in other references as well. Also when comparing the results between the two models, it appears that there are comparisons for two shell/shell models, but these models do not match and no explanation is provided. However the authors highlight that the models predict the measured response with high accuracy (what defines this high accuracy). It would be worth while to see the differences between each of the models if they are included along with some discussion on why they are different.**

Analysis results and description related to ABAQUS were removed from this manuscript. (Comments from Reviewers # 1 and #2)

**Pg 7 – line 1 – the distribution factor is determined by taking the ratio of the moment or shear in the interior girder divided by the sum of the moment or shear in all girder. There is no information provided on whether this is the composite (including moment in deck and axial forces in deck and girder) or noncomposite (moment in girder only) analysis. This should be highlighted or clarified and it should be noted that the results from the two can differ (see *Assessment of flexural lateral load distribution methodologies for stringer bridges in Engineering Structures, Volume 32, Issue 11, November 2010,Pages 3443-3451*). This is a major issue with this manuscript.**

The analyses performed were the composite (including moment in deck and axial forces in deck and girder) analyses. The sentence “The moments were calculated including the moments in the girders and the axial forces in the deck and girders to consider composite behavior of the deck and girders.” was added to clarify this issue. (Page 7, Line 14)

**Pg 7 –line 3 – for a single lane loaded scenario the multiple presence factor is 1.20 not 1.0. Is this what the authors intends (two lanes loaded = 1.0) or was the intent to use the one lane loaded scenario. This could significantly affect results if improperly used.**

The multiple presence factors for the single lane and dual lane trailers were assumed to be 1.00 in this research.

The purpose of using the multiple presence factor of 1.2 for single lane loading in AASHTO LRFD bridge design manual (2009) is to consider that a single vehicle on the bridge can be heavier than each one of a pair of vehicles and still have the same probability of occurrence. This does not appear to be appropriate for the overload vehicles since the weight of the overload vehicles are well measured and controlled and there is minimal chance that the vehicle is 20% heavier than the measured value. Therefore, it seems to be reasonable to assume the multiple presence factors to be 1.00 for the overload vehicles. Please refer the following commentary (C3.6.1.1.2) in AASHTO LRFD bridge design manual (2009)

“The entry greater than 1.0 in Table 1 results from statistical calibration of these Specifications on the basis of pairs of vehicles instead of a single vehicle. Therefore, when a single vehicle is on the bridge, it can be heavier than each one of a pair of vehicles and still have the same probability of occurrence.”

The sentence was revised as follows to clarify this issue, “The multiple presence factor in the AASHTO LRFD bridge design manual (2009) was assumed to be 1.00 to calculate the load distribution factor in this study assuming that the weights of the superload vehicles are well measured and controlled.”. (Page 7, Line 16)

**Pg 7 – line 8 – Figure 6 adds nothing to the paper and should be removed.**

It was removed.

**Pg 7 – line 13 – change “to develop” to “determine”**

It was changed. (Page 8, Line 3)

**Pg 7 –line 14 – this also describes with and without end diaphragms**

The sentence was replaced with “The analysis results for the case-8 bridge configurations in Table 2 with different skews and with and without end diaphragms are shown in the plots of Fig. 6 (moment load distribution factors) and Fig. 7 (shear load distribution factors).” (Page 8, Line 3)

**Pg 7 –line 14 – “The analysis results for the shear load distribution factors for the case-8 bridge...”**

The sentence was replaced with “The analysis results for the case-8 bridge configurations in Table 2 with different skews and with and without end diaphragms are shown in the plots of Fig. 6 (moment load distribution factors) and Fig. 7 (shear load distribution factors).” (Page 8, Line 3)



**Pg 7 – line 15 – add “For all scenarios the load distribution factors decrease...”**

It was added. (Page 8, Line 6)

**Pg 7 – line 17 – add comma after “diaphragms,”**

It was added. (Page 8, Line 10)

**Pg 7 –line 18 – “diaphragms”**

It was replaced. (Page 8, Line 10)

**Pg 7 –line 19 – remove last sentence “The results were...”**

It was removed.

**Pg 8 –line 1 – consider changing multi span to continuous span if this is what is meant**

It was changed. (Page 8, Line 14)

**Pg 8 – line 7 – the term significant is used without justification, this needs to be defined**

The sentence was replaced with “The positive moment load distribution factors for 2 span bridges in Fig. 8 showed less than 6.8% difference compared with those of single span bridges, while the negative moment load distribution factors of 2 span bridges were 23% lower to 52 % higher than the positive moment load distribution factors of the single span bridges.” (Page 8, Line 19)

**Pg 8 – line 8 – change “2 span bridge were -23% lower to 52% higher...” to “2 span bridges were between 23% lower and 52% higher...”**

It was changed. (Page 8, Line 21)

**Pg 8 –line 9 – remove last sentence “The results were...”**

It was removed.

**Pg 8 –line 17 – why were these cases selected, this is not clear**

These cases cover the following configurations of the bridges with concrete I girders. (Page 9, Line 5)

Case ID	Span (m)	Girder spacing (mm)	Deck depth (mm)
19	15.2	2438	229
2	24.4	2438	229
8	36.6	2438	229
20	45.7	2438	229

21	15.2	3353	305
6	24.4	3353	305
22	36.6	3353	305
23	45.7	3353	305

**Pg 8 – line 19 – “comparisons to the same cases without ...”**

It was replaced. (Page 9, Line 8)

**Pg 8 – line 21 – “...while the shear load distribution factors are more dependent on span length than on diaphragms.” Is the case that the DFs are not dependent on end diaphragms?**

The sentence was written to describe that the comparisons of the shear load distribution factors with and without end diaphragm did not show any certain tendency to say that one of the cases is higher or lower since the result is dependent on the span length. The sentence was replaced with “The results show that the moment load distribution factors are larger without end diaphragms while the shear load distribution factors are dependent on the span length.” to clarify this. (Page 9, Line 9)

**Pg 9 –line 1 – 7.7~-7.0% - this does not make sense**

It was replaced with “The shear force load distribution factors found from analysis without end diaphragms were between 7.0 % lower to 7.7 % higher than those with end diaphragms.” (Page 9, Line 13)

**Pg 9 – line 5 – “A complete representation of the effects...” should be highlighted near the front of the investigation discussion. It should also be briefly described if this is any different than what is presented in the manuscript.**

The sentence was provide to inform that only representative results were presented in this paper due to the space limit but all the results can be found elsewhere.

The sentence was replaced with “Representative analysis results are presented in this paper and complete analysis results are presented in a separate report (Bae and Oliva 2010).” and it was moved to the end of the introduction chapter. (Page 4, Line 6)

**Pg 9 – line 15 – “...curve fitting with the analysis data...” - This section needs much more description than is presented in the manuscript (currently none). This is not a trivial matter and the authors should consider referring to NCHRP Report 12-26 for reference. The factor really come out of nowhere with no description on how they were derived. This is a major issue with this manuscript.**

The sentence “The new equations were developed in a manner to ensure that 95% of the predicted

load distribution factors would not be less than those obtained from FEM analysis, i.e. on the safe side.” was added to clarify this issue. (Page 10, Line 17)

**Pg 9-10 – lines (20-21 and 1-2) check format of variable they appear to be superscripted.**

They were checked and revised.

**Pg 10 – line 13 – change “insure” to “ensure”**

It was changed. (Page 10, Line 17)

**Pg 11 – line 17 – change to “No end diaphragms were used in the ...”**

It was changed. (Page 12, Line 3)

**Pg 11 – line 18 – Figures 14 and 15 should include AASHTO comparisons**

They were included in Figure 13 (previously Figure 14).

It seems to be odd to include AASHTO comparisons for the dual lane trailer loading cases in Figure 14 (Previously Figure 15) since there is a large difference in lateral spacing of the wheels. The lateral wheel spacing of the AASHTO two lane loading is 6 ft + 4 ft + 6ft and the lateral wheel spacing of the dual lane loading considered in the Figure is 4 ft + 6ft + 4 ft. Therefore, the AASHTO comparisons for Figure 14 (Previously Figure 15) were omitted.

**Pg 11 – line 21 – the last statement is not clear without AASHTO equation comparisons.**

The comparisons were provided for Figure 13 (previously Figure 14)

**Pg 11 – lines 21-22 – change to “... the developed equations have a wide range of applicability.” (pending previous comment comparison).**

It was changed. (Page 12, Line 8)

**Pg 12 – line 19 – “... an actual bridge load test...” This seems to be referring to midspan deflections, was there any consideration of shear locations (strain measurements). This comment really goes back to the validation process that needs to be revisited and discussed in more depth.**

More details on the model verification were also provided as follows.

“The superstructure of the bridge consists of two continuous spans having a length of 32.81 m. The cross section consists of five prestressed concrete girders equally spaced, supporting a reinforced concrete deck. The bridge was subjected to 2 lane truck loading at the mid-span of the first span and deflections of the five girders were measured at the mid-span of the first span.” (Page 6, Line 11)

Exact verification of the shear results would be desirable, but shear force distribution is much more difficult to measure than flexure since loads for creating shear are placed near the supports and deflection of girders will not be large enough to accurately produce distribution factors. Load cells would be needed at the bearings of the girders. Data from these types of tests were not available.

**Pg 13 – line 12 – the term rationally is used here, but this is somewhat misleading considering that the equations were developed using curve fitting techniques (which were not described in sufficient depth) on top of equations that were already developed using curve fitting techniques (AASHTO equations).**

“Rationally” is not a good term. “Empirically” is a more appropriate term and it has been changed in the text. (Page 13, Line 20)

New equations without considering the AASHTO eq. were developed.

**Pg 14 – line 11 – is this the appropriate way to reference C.O. Hays Jr. (see ASCE style guide)**

It was replaced with “Hays, C. O.” (Page 14, Line 22)

**Pg 17 – Table 1 – The heading “Ranges” is not really an appropriate title (example of ranges = 4-20). Consider changing to something like parameters. Also for the girder type, the four types should be listed**

They were revised. (Table 1)

**Pg 18 – Table 2 – Why is Case ID 7 listed a second time in the table (under set 3).**

It was removed. (Table 2)

**Pg 22 – Fig 4 – consider showing the total load for each of the scenarios after the figure captions [e.g. a) Single land loading case 01 (Total Load = XX kN)]**

The gross weights of the vehicles were provided. (Fig. 4)

**All figures – should not have titles on figures. All information should be described in the captions and may require sub-captions (a) and (b) – Remove text such as Single Lane (Moment): L = 36.6m, S=2438 mm, t = 229mm in Fig 7 (see also Figures 8, 9, 10, 11,13, 14, 15). Also use consistent scales for comparative analyses. Define the units for GDF (e.g. lanes/beam is pretty typical).**

They were revised.

**Reviewer #3: This is a very well written manuscript. However, some revision is needed to address the following comments:**

**1) The authors should compare the models using the girder stresses/strain since load distribution factor (LDF) is more closely related to the girder stresses/strain than deflection.**

It is true that the comparison of the girder stress/strain is a better approach to validate the FE model used to find load distribution factors. Use of strain data, which is often measured in bridge load tests, was found to be very difficult since the strain data appeared to be very dependent on the non-homogeneous nature of the concrete girders- i.e. micro-cracking, aggregate near the surface, and defects in the concrete matrix. The strain data measured in numerous bridges did not even follow the normal Bernoulli hypothesis of plane section behavior in flexure. For these reasons the deflection data was preferred.

It was judged that the purpose of the comparison to validate the finite element analysis techniques was fulfilled from the accuracy of the deflection comparison (less than 8% difference).

**2) Can the author provide the LDF obtained experimentally? It would be good to compare the LDF directly rather than simply comparing to FEM.**

Unfortunately the experimental data for the overload is not available at this time. The LDF behavior of the bridge that was tested was derived from the measured deflection results – so use of the deflection comparison directly seemed most appropriate.

**3) The proposed equations are based on regression analysis using the selected configuration sets of bridges in Table 1. The question is how reliable are the equations for bridges that fall out of the ranges in Table 1, especially for 3 or more spans bridges?**

The ranges in Table 1 were selected to cover the most of the most common bridge configurations. It is judged that the equations are applicable to a bridge with 3 or more spans since it is conservatively commented that “The lateral load distribution obtained for simple spans is also considered applicable to continuous structures.” in the AASHTO LRFD bridge design manual C4.6.2.2.2b (2009).

**4) The constants and exponents shown in Table 3 indicate very high level of accuracy. If the equations are calibrated to be 113% off the FEM, is the level of significant figures necessary?**

The significant figures of the constants and exponents were reduced to make the equations as simple as possible. (Table 3)

**5) In Fig 12, for higher LDF values (> 0.6) there are significant variations between the equations and the FEM, where the equations are becoming more conservative. Could the**

**authors comment on why?**

The higher LDF values with significant variations are from the shear dual lane loading case for the bridge with the longest or shortest span. It seems that the proposed shear LDF equation for the dual lane loading case becomes more conservative near the boundary of the considered span. (Fig. 11)

**6) How are the proposed equations differ in accuracy from other equations proposed by other researchers? The manuscript would be a lot stronger if the authors can prove that the proposed equations are better than the equations that have been proposed by other researchers.**

The proposed equations are the only equations for the super overloads that we found which cover skewed bridges and dual lane trailer vehicles. The only equations for overloads from other researchers (Tabsh and Tabatabai 2001) are the equations for bridges without skew subjected to single lane trailer and the results from those equations are provided in Fig. 12.